Pairing instability in a nematic Fermi liquid

2004 J. Phys.: Condens. Matter 16 3139

(http://iopscience.iop.org/0953-8984/16/18/015)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **16** (2004) 3139–3145 PII: S0953-8984(04)76364-X

Pairing instability in a nematic Fermi liquid

Yong Baek Kim and Hae-Young Kee

Department of Physics, University of Toronto, Toronto, ON, M5S 1A7, Canada

Received 16 February 2004
Published 23 April 2004
Online at stacks.iop.org/JPhysCM/16/3139
DOI: 10.1088/0953-8984/16/18/015

Abstract

Strong interaction between fermions may lead to a novel quantum ground state where the rotational symmetry is spontaneously broken while the translational symmetry is still preserved. Such a 'nematic' Fermi liquid has a spontaneously deformed Fermi surface and supports a gapless Goldstone mode associated with the broken rotational symmetry. We consider a nematic Fermi liquid of fermions carrying two internal quantum numbers that, for example, may represent the layer degree of freedom of a bilayer system. It is found that the effective interaction mediated by a collective mode gives rise to a pairing instability of the underlying fermions under certain conditions, leading to the conclusion that the nematic order could be useful for superconductivity.

1. Introduction

The occurrence of novel anisotropic metallic states in strongly correlated electron systems has been uncovered by the discoveries of stripe phases in cuprates [1, 2], and anisotropic metallic states in high Landau levels of two-dimensional electron systems [3–5]. It has been suggested that the anisotropic states in these systems arise due to strong electron–electron interactions. A primary candidate for such a state has been the 'nematic' Fermi liquid where the rotational symmetry is spontaneously broken while the translational symmetry is still preserved [6–9]. There exist at least two different approaches to get the nematic Fermi liquid. One may first consider a *less-symmetric* smectic state where both the rotational and translational symmetries are spontaneously broken [6, 7]; then the melting of the smectic state via the proliferation of appropriate topological defects will lead to a nematic Fermi liquid phase. Another approach is to introduce a novel interaction in a *more-symmetric* isotropic fermion system such that only the rotational symmetry will be spontaneously broken upon tuning of the interaction.

The smectic state can be obtained by allowing small fluctuations about a unidirectional crystalline phase [6–9]. It is important to notice that there exist low energy fermionic excitations in electronic smectic states in contrast to the classical liquid crystals. These low excitations reside, for example, along the stripes in cuprates or the edges of two adjacent integer quantum Hall states in high Landau levels of two-dimensional electron systems. The existence of low energy fermionic degrees of freedom adds a major difficulty in describing the melting of the smectic state to a nematic Fermi liquid state. It is clear that the nematic order parameter will be strongly affected by the low energy fermionic excitations. So far, no well-controlled procedure has been discovered along this line.

On the other hand, the second approach has proved to be useful for certain purposes. Oganesyan *et al* [10] introduced a two-body interaction between quadrupolar densities of spinless fermions. Upon tuning the interaction strength, the rotational symmetry is spontaneously broken and the nematic order develops. The Goldstone mode of the nematic state is the director mode that corresponds to the angular oscillation in the principal axis of the elongated Fermi surface. Its dispersion relation is highly anisotropic in contrast to the cases of the broken spin symmetry. The interaction between the fermions and director mode leads to an anisotropic self-energy of the quasiparticles, that show non-Landau Fermi liquid properties in certain directions. That is, the lifetime of the quasiparticles strongly depends on the angular directions.

In this paper, we follow the second approach and investigate a pairing instability in a model of a nematic Fermi liquid. In particular, we are interested in finding a toy model where the spontaneous breaking of the rotational symmetry is useful for pairing instability or superconductivity. A somewhat related question has been addressed in the context of the stripes in high T_c cuprates. The toy model we consider may find a useful and more realistic application in cold fermionic atomic gases in optical traps even if its direct application to solid state systems may seem elusive.

More specifically, we consider a fermionic system with two internal quantum numbers that may represent the layer degrees of freedom of a bilayer system or the generalized/pseudo spin degrees of freedom. Using a generalized interaction between quadrupolar densities of the fermions with two internal quantum numbers, we find a nematic Fermi liquid phase where there exist two director modes that correspond to the in-phase and out-of-phase modes of the angular oscillations of the two deformed Fermi surfaces.

In the 'singlet' Cooper channel, the in-phase mode mediates a repulsive interaction between fermions while the out-of-phase mode leads to an attractive interaction. The out-of-phase (in-phase) mode is gapped (gapless), thus normally the repulsive interaction wins in the low energy limit. It may, however, be possible to gap out the in-phase mode independently by external means. In this case, it is found that the combined interaction from two director modes can be made attractive under certain conditions. The resulting paired state has an unusual pairing amplitude that vanishes non-analytically at certain angular directions in momentum space, but it does not change its sign.

2. Choice of the Hamiltonian

We consider the following Hamiltonian for a two-dimensional fermion system with two internal quantum numbers, $\uparrow$ and $\downarrow$.

$$H = \int \mathrm{d}^2 r \sum_{\alpha} \hat{\Psi}^{\dagger}_{\alpha}(\mathbf{r})\epsilon(-\mathrm{i}\nabla)\hat{\Psi}_{\alpha}(\mathbf{r}) + \int \mathrm{d}^2 r\, \mathrm{d}^2 r'\, F_2(\mathbf{r}-\mathbf{r}') \sum_{\alpha} \mathrm{Tr}[\hat{Q}_{\alpha}(\mathbf{r})\hat{Q}_{\alpha}(\mathbf{r}')] + \int \mathrm{d}^2 r\, \mathrm{d}^2 r'\, G_2(\mathbf{r}-\mathbf{r}') \sum_{\alpha\neq\beta} \mathrm{Tr}[\hat{Q}_{\alpha}(\mathbf{r})\hat{Q}_{\beta}(\mathbf{r}')], \tag{1}$$

where $\alpha, \beta = \uparrow, \downarrow$, $\hat{\Psi}_{\alpha}$ is an annihilation operator of the fermion with 'spin' α, and $\epsilon(-\mathrm{i}\nabla)$ is the kinetic energy operator. Here the matrix quadrupolar density of the fermions is defined as

$$\hat{Q}_{\alpha}(\mathbf{r}) = -\frac{1}{k_{\mathrm{F}}^2}\hat{\Psi}^{\dagger}_{\alpha}(\mathbf{r}) \begin{pmatrix} \partial_x^2 - \partial_y^2 & 2\partial_x\partial_y \\ 2\partial_x\partial_y & \partial_y^2 - \partial_x^2 \end{pmatrix} \hat{\Psi}_{\alpha}(\mathbf{r}). \tag{2}$$

Here $F_2(\mathbf{q})$ and $G_2(\mathbf{q})$ are the interactions between the fermions with the same and different 'spin' quantum numbers, respectively:

$$F_2(\mathbf{q}) = \frac{1}{F_2^{-1} + \kappa_1 q^2}, \qquad G_2(\mathbf{q}) = \frac{1}{G_2^{-1} + \kappa_2 q^2}. \tag{3}$$

The detailed form of the interaction does not matter as far as the short range interaction is concerned.

Using a Hubbard–Stratonovich transformation, we can write the effective Lagrangian as $(\sigma = +, -)$

$$\begin{aligned}\mathcal{L} = \int \mathrm{d}^2 r \sum_\alpha \Psi_\alpha^\dagger(\mathbf{r}, \tau)(\partial_\tau + \epsilon(-\mathrm{i}\nabla) - \mu)\Psi_\alpha(\mathbf{r}, \tau) \\ + \sum_{\sigma=\pm}\sum_{\mathbf{q}} F_2(\mathbf{q})[1 + \sigma g(\mathbf{q})]\,\mathrm{Tr}[Q_\sigma(\mathbf{q}, \tau)N_\sigma(-\mathbf{q}, \tau)] \\ + \sum_{\sigma=\pm}\sum_{\mathbf{q}} F_2(\mathbf{q})[1 + \sigma g(\mathbf{q})]\,\mathrm{Tr}[N_\sigma(\mathbf{q}, \tau)N_\sigma(-\mathbf{q}, \tau)],\end{aligned} \tag{4}$$

where $Q_\sigma(\mathbf{q}, \tau) = Q_\uparrow(\mathbf{q}, \tau) + \sigma Q_\downarrow(\mathbf{q}, \tau)$ with

$$Q_\alpha(\mathbf{q}) = \sum_{\mathbf{k}} \Psi_{\alpha,\mathbf{k}-\mathbf{q}}^\dagger \begin{pmatrix} \cos(2\theta_{\mathbf{k}}) & \sin(2\theta_{\mathbf{k}}) \\ \sin(2\theta_{\mathbf{k}}) & -\cos(2\theta_{\mathbf{k}}) \end{pmatrix} \Psi_{\alpha,\mathbf{k}}, \tag{5}$$

and $\theta_{\mathbf{k}}$ is the angle between $\mathbf{k}$ and the x-axis. Here $g(\mathbf{q}) = G_2(\mathbf{q})/F_2(\mathbf{q})$ and $N_\sigma(\mathbf{q}, \tau)$ is the Hubbard–Stratonovich matrix field given by

$$N_\sigma(\mathbf{q}, \tau) = \begin{pmatrix} N_{\sigma,1}(\mathbf{q}, \tau) & N_{\sigma,2}(\mathbf{q}, \tau) \\ N_{\sigma,2}(\mathbf{q}, \tau) & -N_{\sigma,1}(\mathbf{q}, \tau) \end{pmatrix}. \tag{6}$$

Notice that $N_\sigma^{\mathrm{sp}}(\mathbf{q}) = \langle \hat{Q}_\sigma(\mathbf{q}) \rangle$ at the saddle point.

3. Saddle point and Gaussian fluctuations

Now one can integrate out the fermionic degrees of freedom and get an effective action in terms of $N_\sigma(\mathbf{q})$. In the putative nematic liquid state, there should be a uniform saddle point solution $N_\sigma^{\mathrm{sp}} \neq 0$. Notice that non-zero $N_{+,1}^{\mathrm{sp}}$ ($N_{-,1}^{\mathrm{sp}}$) will lead to the symmetric (antisymmetric) distortions of the $\uparrow$ and $\downarrow$ Fermi surfaces. The finite value of $N_{+,2}^{\mathrm{sp}}$ ($N_{-,2}^{\mathrm{sp}}$) determines the amount of the symmetric (antisymmetric) rotations of the principal axis of the $\uparrow$ and $\downarrow$ Fermi surfaces. The saddle point solution depends on the coefficient of the quadratic term in the long wavelength ($\mathbf{q} \to 0$) expansion of the Landau–Ginzburg free energy.

$$\mathcal{F}(N_+, N_-) = M_+\,\mathrm{Tr}[N_+^2] + M_-\,\mathrm{Tr}[N_-^2] + \cdots, \tag{7}$$

where $\cdots$ represents the quartic and higher order terms. Following [10], we assume that the one-particle dispersion has the form of $\epsilon(k) - \mu = v_{\mathrm{F}} q(1 + a(q/k_{\mathrm{F}})^2)$ with $q = |\mathbf{k}| - k_{\mathrm{F}}$ and $a > 0$; then the coefficient of the quartic term can be shown to be positive. One can show that

$$M_\sigma = 2(1 + \sigma g)F_2 + (1 + \sigma g)^2 N(0) F_2^2, \tag{8}$$

where $g = g(\mathbf{q} = 0)$ and $N(0)$ is the density of states at the Fermi level. We will choose $F_2 > 0$ and $G_2 < 0$ such that $M_+ < 0$; then $M_- > 0$ also follows. In this case, the saddle point solution is given by $N_+^{\mathrm{sp}} \neq 0$ and $N_-^{\mathrm{sp}} = 0$ that represent the symmetric distortions and rotations of the $\uparrow$ and $\downarrow$ Fermi surfaces.

The Gaussian fluctuations of $N_\sigma(\mathbf{q}) = N_\sigma^{\mathrm{sp}} + \delta N_\sigma(\mathbf{q})$ about the nematic liquid ground state and their interaction with the fermions can be written as ($a = 1, 2$)

$$\begin{aligned}
\mathcal{L}_{\mathrm{eff}} &= \mathcal{L}_{\mathrm{sp}} + \mathcal{L}_{\mathrm{int}} + \mathcal{L}_{\mathrm{g}}, \\
\mathcal{L}_{\mathrm{int}} &= \sum_{\sigma}\sum_{\mathbf{q}} F_2(1+\sigma g)\,\mathrm{Tr}[Q_\sigma(\mathbf{q},\tau)\delta N_\sigma(-\mathbf{q},\tau)] + \mathrm{c.c.} \\
\mathcal{L}_{\mathrm{g}} &= \sum_{\sigma,a,b}\sum_{\mathbf{q}} \delta N_{\sigma,a}(\mathbf{q},\tau) D^{-1}_{\sigma,ab}(\mathbf{q},\tau)\delta N_{\sigma,b}(-\mathbf{q},\tau),
\end{aligned} \tag{9}$$

where $\mathcal{L}_{\mathrm{sp}}$ describes the mean field nematic liquid ground state. Here the propagators of the Gaussian fluctuations, $D^{-1}_{\sigma,ab}$, are given by

$$D^{-1}_{\sigma,ab}(\mathbf{q}, \mathrm{i}\nu_n) = \Delta_{\sigma,ab} + (1+\sigma g)^2 F_2^2 \Pi_{ab}(\mathbf{q}, \mathrm{i}\nu_n), \tag{10}$$

with $\Delta_{\sigma,ab} = \mathrm{diag}(\Delta_{\sigma,1}, \Delta_{\sigma,2})$ and

$$\Pi_{ab}(\mathbf{q}, \mathrm{i}\nu_n) = N(0)\int \frac{\mathrm{d}\theta}{2\pi}\frac{s}{s-\cos(\theta-\phi)}\begin{pmatrix} 2\cos^2(2\theta) & \sin(4\theta) \\ \sin(4\theta) & 2\cos^2(2\theta)\end{pmatrix}, \tag{11}$$

where ϕ is the angle between $\mathbf{q}$ and the x-axis. Here $\nu_n = 2n\pi T$ is the Matsubara frequency and $s = \mathrm{i}\nu_n/v_{\mathrm{F}}q$. $\Delta_{\sigma,1}$ and $\Delta_{\sigma,2}$ represent the gap in each collective mode, that depend on the interaction strength, but their precise forms are not important in the following analysis. In the nematic liquid state, for generic interactions without an external perturbation, we expect that there should be one gapless Goldstone mode; this is the in-phase director mode represented by $\delta N_{+,2}$, thus $\Delta_{+,2} = 0$.

On the other hand, in the presence of an external quadrupolar potential that couples to the fermions in the symmetric channel, the in-phase mode will acquire a finite gap. This gap would, however, be smaller than that of the out-of-phase mode for a sufficiently weak potential. As will be shown later, both the in-phase ($\delta N_{+,2}$) and out-of-phase ($\delta N_{-,2}$) director modes will be important in the analysis of the pairing instability. We will not consider the gapped distortion modes, $\delta N_{+,1}$ and $\delta N_{-,1}$, in the subsequent analysis.

The propagators of two director modes in the long wavelength and low energy limits can be obtained as

$$D^{-1}_{\sigma,22} = \Delta_{\sigma,2} + \kappa_\sigma q^2 + 2(1+\sigma g)^2 N(0) F_2^2 \frac{|\nu_n|}{v_{\mathrm{F}}q}\sin^2(2\phi), \tag{12}$$

where, in the absence of the external potential, $\Delta_{+,2} = 0$ and $\Delta_{-,2} > 0$. When there is a weak quadrupolar external potential in the symmetric channel, we expect $\Delta_{-,2} > \Delta_{+,2} \neq 0$. Here κ_+ and κ_- are the stiffness coefficients, and their detailed dependence on the interaction strength will not be important.

4. Effective two-body interaction

The fermions with $\uparrow$ and $\downarrow$ quantum numbers interact with each other by exchanging two director modes just like the case of the phonon mediated electron–electron interaction. The effective two-body interaction between fermions can be obtained by integrating out the director modes in the effective action given by equation (9) and using the following form of the coupling between the director modes and the fermions:

$$\sum_{\mathbf{k}}\sum_{\sigma=\pm}(1+\sigma g)\delta N_{\sigma,2}(\mathbf{q})\sin(2\theta_{\mathbf{k}})[\Psi^\dagger_{\uparrow,\mathbf{k}-\mathbf{q}}\Psi_{\uparrow,\mathbf{k}} + \sigma\Psi^\dagger_{\downarrow,\mathbf{k}-\mathbf{q}}\Psi_{\downarrow,\mathbf{k}}]. \tag{13}$$

After integrating out the director modes, the effective interaction between fermions is obtained as

$$S'_{\text{eff}} = \sum_{\mathbf{q},\nu_n}\sum_{\mathbf{k},\omega_m}\sum_{\mathbf{k}',\omega'_m}\sum_{\alpha} V_1(\mathbf{k},\mathbf{k}',\mathbf{q},\mathrm{i}\nu_n)\Psi^{\dagger}_{\alpha,\mathbf{k}-\mathbf{q}}(\omega_m-\nu_n)\Psi^{\dagger}_{\alpha,\mathbf{k}'+\mathbf{q}}(\omega'_m+\nu_n)\Psi_{\alpha,\mathbf{k}'}(\omega'_m)\Psi_{\alpha,\mathbf{k}}(\omega_m)$$
$$+\sum_{\mathbf{q},\nu_n}\sum_{\mathbf{k},\omega_m}\sum_{\mathbf{k}',\omega'_m}\sum_{\alpha\neq\beta} V_2(\mathbf{k},\mathbf{k}',\mathbf{q},\mathrm{i}\nu_n)\Psi^{\dagger}_{\alpha,\mathbf{k}-\mathbf{q}}(\omega_m-\nu_n)\Psi^{\dagger}_{\beta,\mathbf{k}'+\mathbf{q}}(\omega'_m+\nu_n)\Psi_{\beta,\mathbf{k}'}(\omega'_m)\Psi_{\alpha,\mathbf{k}}(\omega_m), \tag{14}$$

where

$$\begin{aligned} V_1(\mathbf{k},\mathbf{k}',\mathbf{q},\mathrm{i}\nu_n) &= F_2^2[(1+g)^2D_{+,22}(\mathbf{q},\mathrm{i}\nu_n)+(1-g)^2D_{-,22}(\mathbf{q},\mathrm{i}\nu_n)][\sin(2\theta_{\mathbf{k}})\sin(2\theta_{\mathbf{k}'})+\sin(2\theta_{\mathbf{k}-\mathbf{q}})\sin(2\theta_{\mathbf{k}'+\mathbf{q}})]\\ V_2(\mathbf{k},\mathbf{k}',\mathbf{q},\mathrm{i}\nu_n) &= F_2^2[(1+g)^2D_{+,22}(\mathbf{q},\mathrm{i}\nu_n)-(1-g)^2D_{-,22}(\mathbf{q},\mathrm{i}\nu_n)][\sin(2\theta_{\mathbf{k}})\sin(2\theta_{\mathbf{k}'})+\sin(2\theta_{\mathbf{k}-\mathbf{q}})\sin(2\theta_{\mathbf{k}'+\mathbf{q}})]. \end{aligned} \tag{15}$$

5. Cooper channel and pairing instability

Notice that V_1 and V_2 correspond to the interactions between the fermions carrying the same and opposite spin quantum numbers, respectively. In the Cooper channel, $\mathbf{k}' = -\mathbf{k}$ with $\theta_{\mathbf{k}'} = \theta_{\mathbf{k}} + \pi$, V_1 is always repulsive, while V_2 can be attractive if $|g| = |G_2|/F_2 > 1$. In the long wavelength (small q) and low energy limit, V_2 in the Cooper channel can be written as (here $\Delta_+ \equiv \Delta_{+,2}$ and $\Delta_- \equiv \Delta_{-,2}$)

$$V_2(\mathbf{k},-\mathbf{k},\mathbf{q},\mathrm{i}\nu_n) \approx -\frac{A\sin^2(2\theta_{\mathbf{k}})}{C + B\frac{|\nu_n|}{v_{\mathrm{F}}q}\sin^2(2\phi)}, \tag{16}$$

where

$$\begin{aligned} A &= 2F_2^2[2|g|(\Delta_+ + \Delta_-) - (|g|^2+1)(\Delta_- - \Delta_+)]\\ B &= 2N(0)F_2^2[(|g|^2+1)(\Delta_+ + \Delta_-) - 2|g|(\Delta_- - \Delta_+)]\\ C &= \Delta_+\Delta_-. \end{aligned} \tag{17}$$

One can show that $V_2 < 0$ if the following condition is satisfied:

$$1 < |g| < \frac{\sqrt{\Delta_-}+\sqrt{\Delta_+}}{\sqrt{\Delta_-}-\sqrt{\Delta_+}}. \tag{18}$$

6. Gap equation and nematic superconductor

Using the attractive interaction V_2, the gap equation for $\Delta(\mathbf{k}) = \langle\Psi_{\uparrow,\mathbf{k}}\Psi_{\downarrow,-\mathbf{k}}\rangle$ can be written as

$$\Delta(\mathbf{k},\mathrm{i}\nu_n) = -T\sum_{\mathbf{k}',\nu'_n}\frac{V_2(\mathbf{k},-\mathbf{k},\mathbf{k}-\mathbf{k}',\mathrm{i}\nu_n-\mathrm{i}\nu'_n)\Delta(\mathbf{k}',\mathrm{i}\nu'_n)}{(\nu'_n)^2+\xi^2_{\mathbf{k}'}+\Delta^2(\mathbf{k}',\mathrm{i}\nu'_n)}. \tag{19}$$

Assuming that $\Delta(\mathbf{k},\mathrm{i}\nu_n)$ depends mostly on the angle $\theta_{\mathbf{k}}$, we can perform the integral over $\xi_{\mathbf{k}'} = \epsilon_{\mathbf{k}'} - \mu$ and get the following results at $T = 0$.

$$\Delta(\theta_{\mathbf{k}},\mathrm{i}\nu) = N(0)\int\frac{\mathrm{d}\theta}{2\pi}\int\frac{\mathrm{d}\nu'}{2\pi}\frac{\Delta(\theta_{\mathbf{k}}+\theta,\mathrm{i}\nu')}{\sqrt{(\nu')^2+\Delta^2(\theta_{\mathbf{k}}+\theta,\mathrm{i}\nu')}}\frac{A\sin^2(2\theta_{\mathbf{k}})}{C+B\frac{|\nu-\nu'|}{v_{\mathrm{F}}k_{\mathrm{F}}|\theta|}\sin^2(2\theta_{\mathbf{k}})}, \tag{20}$$

where $\theta = \theta_{\mathbf{k}} - \theta_{\mathbf{k}'}$. We also use $q \approx k_{\mathrm{F}}|\theta|$ and $\sin^2(2\phi) \approx \sin^2(2\theta_{\mathbf{k}})$ in the long wavelength and low frequency limit. One can easily see that $\Delta(\theta_{\mathbf{k}},\mathrm{i}\nu_n)$ vanishes at $\theta_{\mathbf{k}} = 0, \pm\frac{\pi}{2}, \pi$. It can be shown that the frequency dependence in $\Delta(\theta_{\mathbf{k}},\mathrm{i}\nu_n)$ is weak near $\theta_{\mathbf{k}} = 0, \pm\frac{\pi}{2}, \pi$. When $\Delta(\theta_{\mathbf{k}})\sin^2(2\theta_{\mathbf{k}}) \ll E_{\text{eff}}$, we get

$$\Delta(\theta_{\mathbf{k}}) \approx \frac{E_{\text{eff}}}{\sin^2(2\theta_{\mathbf{k}})}\exp\left(-\frac{1}{N(0)V_{\text{eff}}\sin^2(2\theta_{\mathbf{k}})}\right), \tag{21}$$

where

$$E_{\rm eff} = \frac{2\theta_{\rm c}}{e}\frac{C}{B}v_{\rm F}k_{\rm F}$$
$$V_{\rm eff} = \frac{\theta_{\rm c}}{\pi}\frac{A}{C}. \tag{22}$$

Here $\theta_{\rm c}$ is the cut-off in the integral over the angle θ in equation (20). This is the asymptotically correct solution near $\theta_{\mathbf{k}} = 0, \pm\frac{\pi}{2}, \pi$. Thus the gap is exponentially suppressed in the four directions.

7. Summary and discussion

We study the pairing instability in a class of nematic Fermi liquid state where the rotational symmetry is spontaneously broken due to a novel two-body interactions. In the case of the fermions with no internal quantum number (e.g. spin polarized electrons), the director mode—the Goldstone mode of the nematic order—always mediates a repulsive interaction in the Cooper channel. On the other hand, in a model of fermions carrying internal quantum numbers (e.g. the layer index of a bilayer system or a pseudospin), there exist two independent director modes associated with the in-phase and out-of-phase oscillations of the two deformed Fermi surfaces. It can be shown that the out-of-phase (in-phase) mode mediates an attractive (repulsive) interaction in the Cooper channel. In the absence of an external perturbation, only the in-phase mode is gapless, leading to a net repulsive interaction in the low energy limit. In the presence of an external quadrupolar potential, the in-phase mode also acquires a gap, but it can be smaller than the excitation gap of the out-of-phase mode for a sufficiently weak external potential. In this case, the net effective interaction in the Cooper channel can be attractive under certain conditions; this leads to a pairing instability. The resulting pairing gap has a remarkable structure. There are four nodes where the gap vanishes exponentially, but the gap amplitude does not change sign. Moreover, the angular dependence near the nodes cannot be described by an analytic expansion in terms of the angle around the nodes.

Even though the toy model we considered here may not be directly applicable to realistic systems at the microscopic level, it may appear as a low energy effective theory in certain limits. It is also worthwhile mentioning that our model may find a more direct application in the systems of cold fermionic atoms in optical traps. At the conceptual level, our results may have the following implications in the physics of cuprates and quantum Hall effect.

(1) The occurrence of superconductivity in the stripe phase of cuprates has been a subject of great interest. In particular, the question of whether the stripes (or their dynamic versions) are in favour of or against superconductivity has been controversial [2]. Even though we cannot answer this question here, it is interesting to see that the broken rotational symmetry was useful for pairing instability or superconductivity in our toy model. In fact, it was crucial for getting an effective attractive interaction in this case.

(2) In the lowest Landau level, the Fermi-liquid-like state [11] at $\nu = 1/2$ can be described as a Fermi liquid state of composite fermions [12]. There have been a number of examples of the paired quantum Hall states in the bilayer quantum Hall system, where the ground state can be described as a paired state of the composite fermions [13–15]. In the half-filled high Landau levels, the ground state is the highly anisotropic metallic state [5]. It has been suggested that the nematic Fermi liquid state of the composite fermions may describe the anisotropic metallic state mentioned above [10].

Our work suggests that it may be possible to form an unusual paired quantum Hall state in the half-filled high Landau levels of the bilayer system; that is, under appropriate conditions not presently investigated, a 'nematic' paired quantum Hall state may arise from the pairing instability of the underlying nematic Fermi liquid state of the composite fermions.

Acknowledgments

We thank Steve Kivelson, Leo Radzihovsky, and Vadim Oganesyan for useful discussions. This work was supported in part by the NSERC of Canada, Canadian Institute for Advanced Research, Alfred P Sloan Foundation, and Materials and Manufacturing Ontario.

References

[1] Tranquada J M *et al* 1995 *Nature* **375** 561
[2] See, for example, Emery V J, Kivelson S A and Tranquada J M 1999 *Proc. Natl Acad. Sci.* **96** 8814
Kivelson S A, Fradkin E and Emery V J 1998 *Nature* **393** 550
[3] Lilly M P *et al* 1999 *Phys. Rev. Lett.* **82** 394
[4] Du R R *et al* 1999 *Solid State Commun.* **109** 389
[5] Koulakov A A, Fogler M M and Shklovskii B I 1996 *Phys. Rev. Lett.* **76** 499
Moessner R and Chalker J 1996 *Phys. Rev.* B **54** 5006
[6] Fradkin E and Kivelson S A 1999 *Phys. Rev.* B **59** 8065
[7] MacDonald A H and Fisher M P A 2000 *Phys. Rev.* B **61** 5724
[8] Wexler C and Dorsey A T 2001 *Phys. Rev.* B **64** 115312
[9] Radzihovsky L and Dorsey A T 2001 *Preprint* cond-mat/0110083
[10] Oganesyan V, Kivelson S A and Fradkin E 2001 *Phys. Rev.* B **64** 195109
[11] Halperin B I, Lee P A and Read N 1993 *Phys. Rev.* B **47** 7312
[12] Jain J K 1989 *Phys. Rev. Lett.* **63** 199
[13] Read N and Green D 2000 *Phys. Rev.* B **61** 10267
Moore G and Read N 1991 *Nucl. Phys.* B **360** 362
[14] Bonesteel N E 1993 *Phys. Rev.* B **48** 11484
[15] Kim Y B, Nayak C, Demler E, Read N and Das Sarma S 2001 *Phys. Rev.* B **63** 205315